

Comment on “Which-way information in a nested Mach-Zehnder interferometer”

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Potoček and Ferenczi [Phys. Rev. A **92**, 023829 (2015)] provided an analysis of the experimental evidence obtained by Danan *et al.* [Phys. Rev. Lett. **111**, 240402 (2013)] for the surprising behaviour of photons passing through an interferometer, in particular, motion on disconnected paths. Potoček and Ferenczi reproduced the results of the experiment, but when analyzing its modification, they claimed that the reasoning of Danan *et al.*, which led to disconnected paths, is erroneous. It is argued here that the criticism of Potoček and Ferenczi is unfounded.

Potoček and Ferenczi (PF) presented a very detailed and correct analysis of the experiment by Danan *et al.* [2] in terms of classical optics, significantly extending the analysis of Saldanha [3]. These analyses agree completely with a brief analysis in the framework of classical optics appearing in [2]. PF correctly state that the classical optics analysis explains all the results without the need for disconnected photon paths. Classical optics does not have a concept of a photon, let alone a post-selected photon, so of course, there are no photon paths, connected or not.

The novel results of PF are related to a modification of Danan *et al.* experiment. They correctly show that the relative heights of the peaks in [2] depend on the three path lengths, and, in particular, can be tuned such that, without blocking any of the paths, the trace of only mirror *C* is present in the output signal. The error of PF is their assertion that the lack of signals from mirrors *A* and *B* in their modification of the experiment is of the same kind as the lack of signals from mirrors *E* and *F* in experiment [2].

Quantum mechanics has a concept of a photon, and there is a clear meaning for the location of the wave packet of a (pre-selected) photon. However, standard quantum mechanics, as classical optics, has no definition of the location of a pre- and post-selected photon. Danan *et al.* experiment tested the paths of photons inside the interferometer according to the *definition*, given in [4], as *a place where the photon leaves a weak trace*. The nested interferometer in [2] is surprising, mainly because, for a particular setup, the weak trace definition disagrees with the “common sense” definition of Wheeler [5]: *the photon was present only in the paths through which it could pass*.

Since observing the trace left by the pre- and post-selected photons on other systems is an extremely difficult task, in the experiment [2], a degree of freedom of the photons themselves served as a pointer variable for measuring the trace. This was the transversal momentum of the photon which was read off through the positions of the photons on the quad-cell detector.

The two-state vector formalism [6] (TSVF) describes a pre- and post-selected photon by a two-state vector. The TSVF provides a simple way to characterize the weak trace which the pre- and post-selected photon leaves. It vanishes outside the overlap of the forward and the backward evolving states. The strength of the trace in a particular location is characterized by the weak value of the projection on this location. At the experiment [2], there were non-zero weak values of the projection on the mirrors inside the inner interferometer, in spite of the fact that, due to the interference, the photons passing through the inner interferometer could not reach the detector. The weak values of the projections on different mirrors were given by Eq. (3) of [2]:

$$(\mathbf{P}_A)_w = (\mathbf{P}_C)_w = 1, \quad (\mathbf{P}_B)_w = -1, \quad (\mathbf{P}_E)_w = (\mathbf{P}_F)_w = 0. \quad (1)$$

The weak values of the projections show an additional surprising feature. Although there is a trace inside the inner interferometer, there is no trace leading towards and out of it.

Let us consider now the modification of PF in the framework of the TSVF. To eliminate signals at *A* and *B* they suggest to add a phase to path *C*, equal to the Gouy phase $\zeta(z_D)$, without changing anything else. For simplicity, assume that the detector is far away, so the Gouy phase is $\zeta(z_D) = \frac{\pi}{2}$. Adding phase $\frac{\pi}{2}$ to path *C* changes the two-state vector describing the photon at the intermediate time. Instead of Eq. (1) of [2], it is:

$$\langle \Phi | \Psi \rangle = \frac{1}{\sqrt{3}} (\langle A | + i \langle B | + \langle C |) \quad \frac{1}{\sqrt{3}} (|A\rangle + i|B\rangle + i|C\rangle). \quad (2)$$

At mirrors *E* and *F*, the forward and backward evolving states do not overlap, as in the original case. Then, the weak values of the projections on different mirrors are:

$$(\mathbf{P}_B)_w = -(\mathbf{P}_A)_w = i, \quad (\mathbf{P}_C)_w = 1, \quad (\mathbf{P}_E)_w = (\mathbf{P}_F)_w = 0. \quad (3)$$

These values explain the null result in the quad-cell detector at the frequencies of all detectors except C , but for very different reasons. At frequencies f_E and f_F , the null result is because there is no trace in E and F . At frequencies f_A and f_B , the null result is because the quad-cell detector at large distance provides the real part of the weak value of the projections which happens to be zero. The effect of the imaginary part of the weak value on the measuring device is a shift of the conjugate variable, the lateral shift of the beam, see Eq. (10) of [6]. For large z_D , it is negligible in comparison to the lateral shift due to the change of the direction of the beam. However, when the detector is not very far, the effect of the imaginary part is significant. In fact, in the experiment [2], a serious effort was required to keep the phases stable, to avoid imaginary weak values of the projections.

The null signals at frequencies f_A and f_B in the PF modification of the experiment are not because the photons do not leave a trace at mirrors A and B , but because the measurement procedure, with a particular distance to the detector, fails to detect it. Changing the position of the detector reveals the frequencies f_A and f_B . In contrast, keeping destructive interference towards mirror F ensures null signals at f_E and f_F at any position of the detector.

A subtle point worth repeating [7–9], is that when all mirrors vibrate, none of the signals, including f_E and f_F , are exactly zero. And if we postulate that in the experiment there is a zero trace at E and F , it is impossible to have signals at f_A and f_B . What allows us to say that there is a trace in A and B , but disregard the trace in E and F , is that the ratio between the strengths of the traces becomes arbitrary large at the weak limit, while the ratio between the trace in C and the trace in A and B , remains constant. If all mirrors vibrate with the same amplitude proportional to a small parameter ϵ , then the traces in C , A and B are proportional to ϵ , while the traces in E , and F are proportional to ϵ^2 .

The photons in the interferometer of Danan *et al.* leave a trace which includes a disconnected path. The experiment [2] faithfully shows this (together with a continuous path C). In the PF modification of Danan *et al.* experiment the photons leave a trace with a disconnected path too. The null signals at f_A and f_B in PF's modified experiment arise from an inappropriate method of observing this trace. Therefore, the argument of PF against Danan *et al.* conclusions does not hold.

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- [1] V. Potoček and G. Ferenczi, Which-way information in a nested Mach-Zehnder interferometer, *Phys. Rev. A* **92**, 023829 (2015).
 - [2] A. Danan, D. Farfurnik, S. Bar-Ad and L. Vaidman, Asking photons where they have been, *Phys. Rev. Lett.* **111**, 240402 (2013).
 - [3] P.L. Saldanha, Interpreting a nested Mach-Zehnder interferometer with classical optics, *Phys. Rev. A* **89**, 033825 (2014).
 - [4] L. Vaidman, Past of a quantum particle, *Phys. Rev. A* **87**, 052104 (2013).
 - [5] J. A. Wheeler, The 'Past' and the 'Delayed-Choice Double-Slit Experiment', in *Mathematical Foundations of Quantum Theory*, A.R. Marlow, ed., pp. 948, (Academic Press 1978).
 - [6] Y. Aharonov and L. Vaidman, Properties of a quantum system during the time interval between two measurements, *Phys. Rev. A* **41**, 11 (1990).
 - [7] L. Vaidman, Reply to the Comment on 'Past of a quantum particle', *Phys. Rev. A* **88**, 046103 (2013).
 - [8] H. Salih, Commentary: "Asking photons where they have been" - without telling them what to say, *Front. Phys.* **3**, 47 (2015).
 - [9] A. Danan, D. Farfurnik, S. Bar-Ad and L. Vaidman, Response: Commentary: "Asking photons where they have been" - without telling them what to say, *Front. Phys.* **3**, 48 (2015).